Oxford Cambridge and RSA

Tuesday 3 June 2025 – Afternoon

AS Level Further Mathematics B (MEI)

Y413/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 A network has ten vertices, A to J. The table shows the distances between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I	J
A		12	6		7		8			2
B	12		5	14		9				
C	6	5					10			7
D		14						3		8
E	7					6			4	9
F		9			6					
G	8		10						5	
H				3						11
I					4		5			
J	2		7	8	9			11		

Apply the tabular form of Prim's algorithm to the network, starting at A, to find a minimum spanning tree for the network.

Your solution should contain the following.

- The order in which the arcs are selected
- The total length of the arcs in the minimum spanning tree **[4]**

2 The list below shows the sizes of eleven items.

28 25 19 32 18 22 3 12 20 7 5

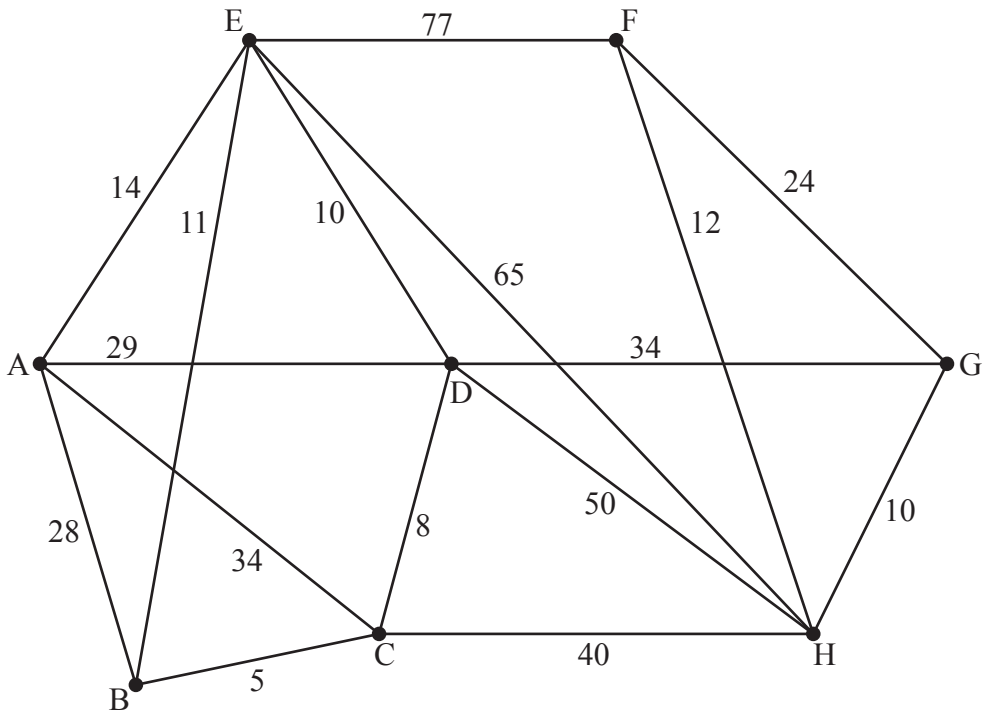
(a) **(i)** Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

(ii) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

(b) A computer takes 4.7×10^{-8} seconds to pack 50 items with sizes 100, 99, 98, … , 53, 52, 51 into bins that have a capacity of 100 using the first fit decreasing algorithm.

Calculate approximately how long it will take the same computer to pack 1 000 000 items with sizes 2 000 000, 1 999 999, 1 999 998, … , 1 000 003, 1 000 002, 1 000 001 into bins that have a capacity of 2 000 000 using the first fit decreasing algorithm. **[2]**

3 The diagram shows a network of roads. The number on each arc represents the length, in miles, of the corresponding road.



Finley, who lives at F, needs to drive to a friend's house at A before they travel together to a campsite situated at C.

Finley decides to use Dijkstra's algorithm **once** to find the shortest route from F to C via A.

(a) Explain why vertex A should be chosen as the starting vertex for the algorithm. **[1]**

(b) **(i)** On the copy of the network in the Printed Answer Booklet, apply Dijkstra's algorithm once to find the length of the shortest route from F to C via A. **[5]**

(ii) State the corresponding shortest route from F to C via A. **[1]**

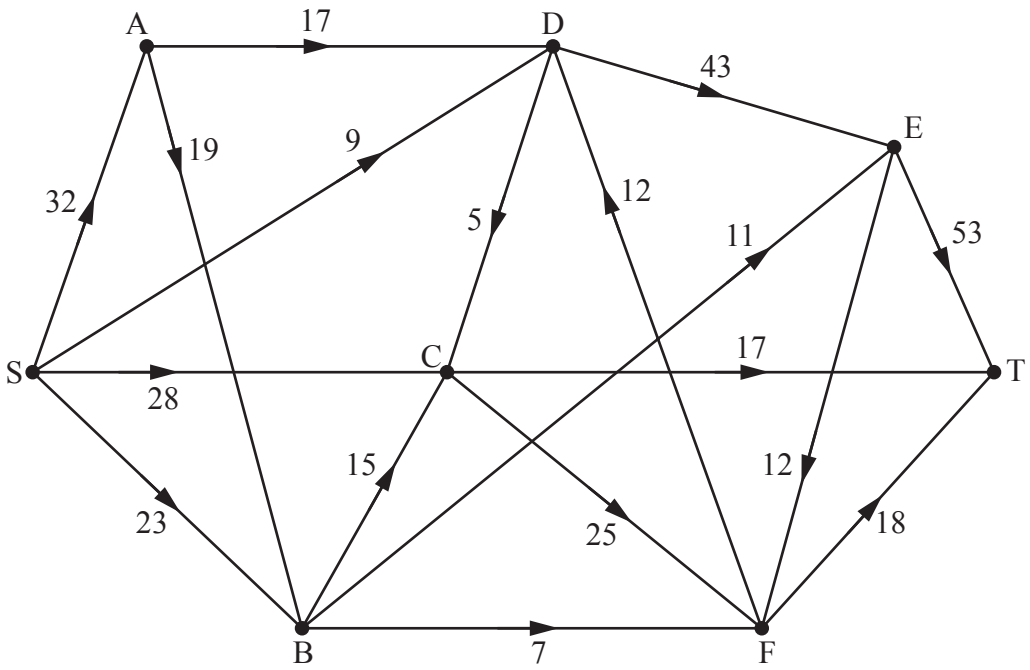
4 **(a)** Determine the number of nodes in the complete graph with 1431 arcs. **[2]**

(b) **In this question you must show detailed reasoning.**

A simply connected graph G has eight nodes of orders 1, 1, 2, 3, 3, 4, 4 and x.

- Determine the **three** possible values of x.
- Use the nodes in the Printed Answer Booklet to draw an example of G for each of these **three** values of x. **[6]**

5 The diagram represents a system of pipes through which water flows continuously from a source S to a sink T. The weights on the arcs show the capacities of the pipes in litres per minute.



(a) Explain why the maximum possible flow along DE must be less than 43 litres per minute. **[1]**

(b) The cut α partitions the vertices into the sets {S, A, B, C, D, F}, {E, T}.

Calculate the capacity of cut α. **[1]**

(c) Explain why partitioning the vertices into sets {S, A, D, E, T}, {B, C, F} does **not** give a cut. **[1]**

An LP formulation is set up to find the maximum flow through the system.

(d) **(i)** Write down the required constraint in the LP formulation regarding the flow through vertex D. **[1]**

(ii) Write down the required constraint in the LP formulation regarding the flow along arc AD. **[1]**

The LP formulation for the system was run in a solver and some of the output is shown in the table.

Variable	Value
SA	32.000 000
SC	28.000 000
SB	15.000 000
BC	12.000 000
DC	0.000 000
DE	38.000 000
ET	49.000 000

(e) **(i)** Complete the diagram in the Printed Answer Booklet to show the maximum flow through the system. **[2]**

(ii) State the maximum value of the flow through the system. **[1]**

(f) Use a suitable cut to prove that this is the maximum flow. **[2]**

The capacity of pipe SD is increased to x litres per minute. In doing so, the maximum flow through the system is increased as much as possible. The capacities of the other pipes in the system remain unchanged.

(g) **(i)** Find the least possible value of x. **[1]**

(ii) State the corresponding maximum possible flow through the system. **[1]**

6 Consider the following LP problem in X and Y.

Maximise $Q = -8X + 9Y$

Subject to

$$\begin{aligned} -5X + 6Y &\leqslant 87 \\ 2X + Y &\leqslant 6 \\ -3X - Y &\leqslant 16 \\ X \leqslant -2, Y &\geqslant 0 \end{aligned}$$

(a) Explain why the simplex algorithm cannot be used to solve this LP problem. **[1]**

The LP problem is reformulated into standard form. The reformulated problem consists of a linear objective function P (that is to be maximised) and constraints in terms of x and y.

The tableau **after** the first iteration of the simplex method, is given in **Table 6.1**.

Table 6.1

P	x	y	s_1	s_2	s_3	RHS
1	−26	0	0	9	0	90
0	17	0	1	−6	0	17
0	−2	1	0	1	0	10
0	1	0	0	1	1	20

(b) State the column that contains the pivot value for the first iteration. You must give a reason for your answer. **[2]**

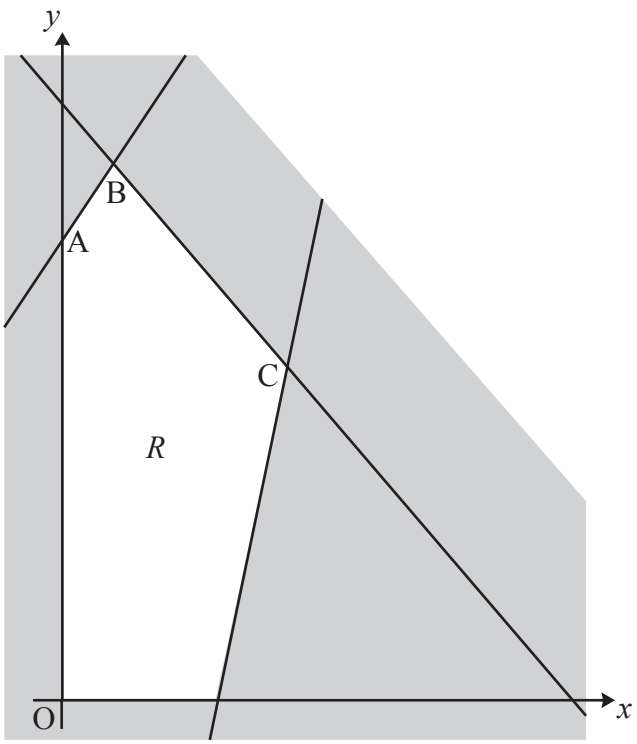
(c) Perform the second iteration of the simplex method. Give each entry as an exact value. **[3]**

The tableau after a third iteration of the simplex method, is given in **Table 6.2**.

Table 6.2

P	x	y	s_1	s_2	s_3	RHS
1	0	0	$\frac{35}{23}$	0	$\frac{3}{23}$	$\frac{2725}{23}$
0	1	0	$\frac{1}{23}$	0	$\frac{6}{23}$	$\frac{137}{23}$
0	0	1	$\frac{3}{23}$	0	$-\frac{5}{23}$	$\frac{181}{23}$
0	0	0	$-\frac{1}{23}$	1	$\frac{17}{23}$	$\frac{323}{23}$

The diagram shows the constraints of the reformulated LP problem. The feasible region, R, is the unshaded region together with its boundaries. Three vertices, A, B, and C, of R are also shown.



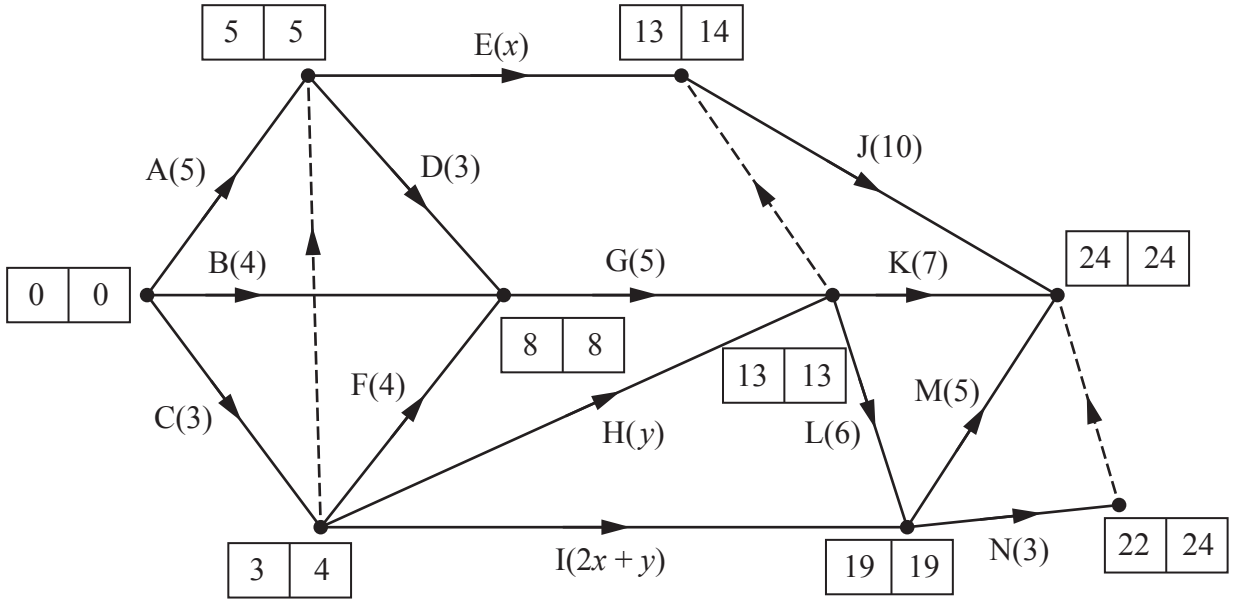
(d) Write down the coordinates of A, B and C. **[3]**

You are given that the reformulation of the original LP into standard form was done using the substitutions $X = -2 - x$, $Y = y$ and $Q = P + 16$.

(e) Determine the maximum value of Q, and the corresponding values of X and Y. **[2]**

Turn over for question 7

7 The diagram shows an activity network for a project. Each activity is represented by an arc. The expressions in brackets show the duration of these activities in hours. The early event times and late event times are shown at each vertex.



(a) Complete the tables in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

(b) **(i)** State the minimum completion time for the project. **[1]**

(ii) State the critical activities of the project. **[1]**

A student is tasked with finding the durations of activities E, H and I. The student knows the following information regarding the activities in the project.

- The sum of the durations of all the activities in the project is at least 79 hours.
- The durations of activities E, H and I must satisfy the early event times and late event times shown in the activity network.
- The combined duration, in hours, of activities E and H must be as small as possible.

(c) Formulate the student's task as an LP problem in x and y. **[4]**

(d) By representing the feasible region for x and y graphically, find the durations of activities E, H and I. **[4]**

END OF QUESTION PAPER

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